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Electromagnetic penguin operators and direct CP violation in $K \to \pi\ell^+\ell^-$

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Abstract: Supersymmetric extensions of the Standard Model predict a large enhancement of the Wilson coefficients of the dimension-five electromagnetic penguin operators affecting the direct CP violation in $K_L \to \pi^0 e^+e^-$ and the charge asymmetry in $K^\pm \to \pi^\pm\ell^+\ell^-$. Here we compute the relevant matrix elements in the chiral quark model and compare these with the ones given by lattice calculations.

Keywords: Beyond Standard Model, Rare Decays, Kaon Physics, CP violation.

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Contents

1. Introduction

Rare kaon decays provide an ideal place both to test the Standard Model (SM) and to unravel new physics beyond it [1]. The origin of CP violation is still an open question in modern particle physics. Dimension-five operators including the electromagnetic and chromomagnetic penguin operators (EMO and CMO) play important roles in these studies since the CP-violating effects from these operators are suppressed in the SM but could be enhanced in its extensions [2]–[6]. In fact present experiments, HyperCP [7] and KLOE [9], and planned ones, NA48b [8], are going to substantially improve the present limits on the Wilson coefficients of these operators by studying CP-violating asymmetries in $K^\pm \to 3\pi$, $K^\pm \to \pi\pi\gamma$ and in $K^\pm \to \pi^\pm\ell\bar{\ell}$ ($\ell = e, \mu$). As we shall see, although it is hard to test the SM now it is possible to probe interesting new physics scenarios. To this purpose it is necessary to know hadronic matrix elements accurately: we address this issue in a particular bosonization scheme.

The weak effective hamiltonian, contributed by EMO and CMO, can be written as [2, 6]

$$\mathcal{H}_{\rm eff} = C_\gamma^+(\mu)Q_\gamma^+(\mu) + C_\gamma^-(\mu)Q_\gamma^-(\mu) + C_g^+(\mu)Q_g^+(\mu) + C_g^-(\mu)Q_g^-(\mu) + \text{h.c.}\,, \tag{1.1}$$

where $C^\pm_{\gamma,g}$ are the Wilson coefficients and

$$Q_\gamma^\pm = \frac{eQ_d}{16\pi^2}\left(\bar{s}_L\sigma_{\mu\nu}d_R \pm \bar{s}_R\sigma_{\mu\nu}d_L\right)F^{\mu\nu}\,, \tag{1.2}$$

$$Q_g^\pm = \frac{g}{16\pi^2}\left(\bar{s}_L\sigma_{\mu\nu}t_a d_R \pm \bar{s}_R\sigma_{\mu\nu}t_a d_L\right)G_a^{\mu\nu}\,. \tag{1.3}$$

Here $\sigma_{\mu\nu} = i/2[\gamma_\mu, \gamma_\nu]$. The SM structure, $\mathrm{SU}(2)_L \times \mathrm{U}(1)$, imposes a chiral suppression for the following operators [10, 11]:

$$\mathcal{H}_{\rm eff}^{\rm SM} = \frac{G_F}{\sqrt{2}} V_{\rm td} V_{\rm ts}^* \Big[C_{11} \frac{g}{8\pi^2} \left(m_d \bar{s}_L \sigma_{\mu\nu} t_a d_R + m_s \bar{s}_R \sigma_{\mu\nu} t_a d_L \right) G_a^{\mu\nu} + \\ + \; C_{12} \frac{e}{8\pi^2} \left(m_d \bar{s}_L \sigma_{\mu\nu} d_R + m_s \bar{s}_R \sigma_{\mu\nu} d_L \right) F^{\mu\nu} \Big] + \text{h.c.}\,, \tag{1.4}$$

and

$$C_{11}(m_W) = \frac{3x^2}{2(1-x)^4} \ln x - \frac{x^3 - 5x^2 - 2x}{4(1-x)^3}\,, \tag{1.5}$$

$$C_{12}(m_W) = \frac{x^2(2-3x)}{2(1-x)^4} \ln x - \frac{8x^3 + 5x^2 - 7x}{12(1-x)^3}\,, \tag{1.6}$$

where $x = m_t^2/m_W^2$ and t_a are the SU(3)-matrices. However, as we shall see, new flavour structures in the supersymmetry-breaking terms allow us to avoid the chiral suppression for the operators in eq. (1.4).

Among rare kaon decays, the flavour-changing neutral current (FCNC) transitions $K \to \pi \ell^+ \ell^-$, induced at the one-loop level in the SM, are well suited to explore its quantum structure and extensions [1, 12, 13]. The decay $K_L \to \pi^0 e^+ e^-$ receives contributions from three sources [1, 14, 15]: direct CP violation, indirect CP violation due to K^0–$\bar{K}^0$ mixing, and CP conservation from the two-photon rescattering in $K_L \to \pi^0 \gamma\gamma$. Therefore, once long-distance effects have been carefully disentangled [15], new physics, induced by the operators in eq. (1.2), can be probed in this channel. Analogously the charge asymmetry in $K^\pm \to \pi^\pm \ell^+ \ell^-$ could be enhanced by a large Wilson coefficient of the operator in eq. (1.2) [16]. Recently it has been shown that also T-odd correlations in charged K_{l4}-decays depend upon the effective hamiltonian in (1.1) [17].

We thus consider here the matrix element $\langle \pi^0 | Q_\gamma^+ | K^0 \rangle$ to determine the observables discussed above. In order to evaluate the bosonization of the EMO we exploit the chiral quark model, which provides an effective link between QCD and low energy chiral perturbation theory. This is particularly interesting since the first lattice calculation of the matrix element $\langle \pi^0 | Q_\gamma^+ | K^0 \rangle$ has been done in ref. [6] and thus a comparison of the two methods can be performed. This might be useful in general to understand the extent of validity of the two approaches in the evaluation of other matrix elements such as the penguin operator.

2. The chiral quark model

The chiral quark model (χQM) [18] has been extensively used to study low energy hadronic physics involving strong and weak interactions [19]–[24]. Note that the interactions among mesons proceeds in this model only by means of quark loops: starting from the short-distance effective hamiltonian in terms of quark operators (such as four-quark operators, EMO, and CMO), the χQM allows us to deduce the low energy effective lagrangian in terms of the input parameters of the model.

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In the χQM [19], a term that represents the coupling between the light (constituent) quarks and the Goldstone mesons

$$- M_Q \left(\bar{q}_R U q_L + \bar{q}_L U^+ q_R\right) \tag{2.1}$$

has been introduced into the QCD lagrangian. The Goldstone meson fields, $\phi(x)$, are collected in a unitary 3×3 matrix $U = \exp(i/f_\pi \lambda \cdot \phi(x))$ (where the λ^a's are the 3×3 Gell-Mann matrices and $f_\pi \simeq 93\,\mathrm{MeV}$) with $\det U = 1$, which transforms as

$$U \longrightarrow V_R U V_L^+ \tag{2.2}$$

under chiral $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ transformations (V_L, V_R), and

$$\frac{1}{\sqrt{2}} \lambda \cdot \phi(x) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix} . \tag{2.3}$$

In the presence of the term (2.1), it is convenient to use new quark fields, Q_L and Q_R, called "rotated basis", defined as follows

$$\begin{aligned} Q_L &= \xi q_L \,, \qquad \bar{Q}_L = \bar{q}_L \xi^+ \,, \\ Q_R &= \xi^+ q_R \,, \qquad \bar{Q}_R = \bar{q}_R \xi \,, \end{aligned} \tag{2.4}$$

with ξ chosen such that

$$U = \xi^2 \,. \tag{2.5}$$

The chiral $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ transformation

$$\xi(x) \longrightarrow V_R \xi(x) h^+(x) = h(x) \xi(x) V_L^+ \tag{2.6}$$

defines the compensating $\mathrm{SU}(3)_V$ transformation $h(\phi(x))$, which is the wanted ingredient for a non-linear representation of the chiral group. Then $Q_{L,R}$'s transform as

$$Q_L \longrightarrow h(x) Q_L \,, \qquad Q_R \longrightarrow h(x) Q_R \,, \tag{2.7}$$

while the term (2.1)

$$-M_Q \left(\bar{q}_R U q_L + \bar{q}_L U^+ q_R\right) = -M_Q \left(\bar{Q}_R Q_L + \bar{Q}_L Q_R\right) \tag{2.8}$$

is invariant. Therefore, the quark fields $Q_{L,R}$ can be interpreted as "constituent chiral quarks" and M_Q as a "constituent quark mass".

Now in order to evaluate the bosonization of the EMO, we firstly write down the EMO using the "rotated basis" in the euclidean space

$$\begin{aligned} \mathcal{H}_{\mathrm{eff}}^{\mathrm{SM}} = {} & \bar{Q} \left(\frac{1-\gamma_5}{2} \xi^+ \lambda \xi^+ m_s + \frac{1+\gamma_5}{2} \xi \lambda \xi m_d \right) \sigma_{\mu\nu} Q C_{\mathrm{EMO}} F^{\mu\nu} + \\ & + \bar{Q} \left(\frac{1+\gamma_5}{2} \xi \lambda^+ \xi m_s + \frac{1-\gamma_5}{2} \xi^+ \lambda^+ \xi^+ m_d \right) \sigma_{\mu\nu} Q C_{\mathrm{EMO}}^* F^{\mu\nu} \,, \end{aligned} \tag{2.9}$$

where $\lambda_{ij} = \delta_{i3}\delta_{j2}$, and

$$C_{\rm EMO} = \frac{G_F}{\sqrt{2}}\frac{e}{8\pi^2}\lambda_t C_{12}\,, \qquad \lambda_t = V_{\rm td}V_{\rm ts}^*\,. \tag{2.10}$$

Here we use the form of EMO in the SM (eq. (1.4)). It is very easy to extend it to the general form in eq. (1.1).

Then the effective action induced by the EMO can be written as follows

$$\Gamma_E(\mathcal{A}, M) = -\frac{1}{2}\int d^4x\,{\rm Tr}\int_0^\infty \frac{d\tau}{\tau}\int \frac{d^dp_E}{(2\pi)^d}\exp\left[-\tau(p_E^2 + M_Q^2)\right]\exp(-\tau\mathcal{D}')\,, \tag{2.11}$$

where Tr is the trace over colour, flavour and Lorentz space, $\mathcal{D}'$ is defined in (A.24), and the integral over τ is introduced by using the proper time method [25]. The detailed derivation for eq. (2.11) has been shown in the Appendix, and dimensional regularization has been used for the involved divergences. Expanding $\exp(-\tau\mathcal{D}')$ in powers of τ, and integrating over the momenta, one can get the effective action in powers of τ, and the corresponding coefficients are the so-called Seeley–DeWitt coefficients. Then the effective lagrangian can be obtained by integrating out τ. The standard procedure can be found in refs. [25, 19]. If we set $F_1 = F_2 = J_{\mu\nu} = 0$ in $\mathcal{D}'$ (see (A.24) in Appendix), which implies that the EMO is switched off, eq. (2.11) will give the same effective lagrangian as in ref. [19]. Here we are concerned about the effective lagrangian generated from the EMO, which is relevant to $K \to \pi\ell^+\ell^-$ transitions. Thus at the leading order we get

$$\mathcal{L}_{\rm EMO}^{\rm SM} = \frac{iN_C M_Q}{8\pi^2}C_{\rm EMO}\left\langle m_d\lambda U L_\mu L_\nu + m_s\lambda L_\mu L_\nu U^+\right\rangle F^{\mu\nu} + {\rm h.c.}\,, \tag{2.12}$$

where $L_\mu = iU^+D_\mu U$, N_C is the number of colours, and $\langle A\rangle$ denotes the trace of A in the flavour space. Likewise, the corresponding effective lagrangian from the general form of the EMO in eq. (1.1) is

$$\mathcal{L}_{\rm EMO}^{\pm} = \frac{iN_C M_Q}{8\pi^2}\frac{eQ_d}{16\pi^2}C_\gamma^{\pm}\left\langle \lambda U L_\mu L_\nu \pm \lambda L_\mu L_\nu U^+\right\rangle F^{\mu\nu} + {\rm h.c.}\,, \tag{2.13}$$

where $\mathcal{L}_{\rm EMO}^{+}$ ($\mathcal{L}_{\rm EMO}^{-}$) generates parity-even (odd) transitions.

The matrix elements of the EMO between a K^0 and a π^0 can be written as

$$\left\langle\pi^0|Q_\gamma^+|K^0\right\rangle = i\frac{\sqrt{2}eQ_d}{16\pi^2 m_K}p_\pi^\mu p_K^\nu F_{\mu\nu}B_T\,, \tag{2.14}$$

$$\left\langle\pi^0|Q_\gamma^-|K^0\right\rangle = 0\,. \tag{2.15}$$

Then from eqs. (1.1) and (2.13), we can obtain

$$B_T = \frac{N_C M_Q m_K}{4\pi^2 f_\pi^2}\,. \tag{2.16}$$

Setting M_Q =0.3 GeV, we have $B_T = 1.31$, which is consistent with $B_T = 1.18 \pm 0.09$ found in the lattice [6] and $B_T \simeq 1$ in ref. [10], and the range $|B_T| = 0.5 \sim 2$ adopted in ref. [2]. Our theoretical error on B_T in (2.16) has two sources: i) from the quark mass M_Q,

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which we believe it is very small, $\sim 10\%$, and ii) from higher order corrections in the χQM, generated by large-N_c gluonic interactions. We have evaluated this contribution using the standard techniques in refs. [19, 20, 26], finding the correction to (2.16)

$$\frac{\pi^2}{9N_c}\frac{\langle \frac{\alpha_s}{\pi} GG\rangle}{M_Q^4}. \tag{2.17}$$

The size of the gluon condensate cannot be simply related to the one which appears in the QCD sum rule [26]. However terms like the one in (2.17), but with larger coefficients, correct also the leading order predictions for the L_i's and f_π [19]. Model consistency and the phenomenologically successful predictions of the leading order evaluation, lead us to the reasonable expectation that the gluon correction in (2.17) cannot exceed $\sim 30\%$ and so consequently we can very conservatively estimate the error in this way on B_T, i.e. $B_T = 1.31 \pm 0.4$.

We stress that the agreement with the lattice is found for natural values of the chiral quark model. So we can be quite confident in this result.

3. $K \to \pi\ell^+\ell^-$

The decay width of $K_L \to \pi^0 e^+e^-$ induced by the EMO is given by

$$\mathrm{Br}(K_L \longrightarrow \pi^0 e^+ e^-)_{\mathrm{EMO}} = 8.9 \times 10^3\,\mathrm{GeV}^2 B_T^2 \left|\mathrm{Im}\, C_\gamma^+\right|^2. \tag{3.1}$$

To obtain an interesting bound on $\mathrm{Im}\, C_\gamma^+$ we improve our error on B_T by considering also the lattice results [6]. Thus from the experimental upper bound [27]

$$\mathrm{Br}(K_L \longrightarrow \pi^0 e^+ e^-) < 5.1 \times 10^{-10}, \tag{3.2}$$

we get

$$\left|\mathrm{Im}\, C_\gamma^+\right| < 1.8 \times 10^{-7}\,\mathrm{GeV}^{-1} \tag{3.3}$$

at 80% C.L.

It is known that $K^\pm \to \pi^\pm\ell^+\ell^-$ is dominated by long-distance, charge-symmetric, one-photon exchange [12, 28, 29, 30]. This piece can be written as [30]

$$A(K^+ \longrightarrow \pi^+\ell^+\ell^-) = -\frac{e^2}{m_K^2(4\pi)^2} W_+(z)(p_K + p_\pi)^\mu \bar{u}(p_-)\gamma_\mu v(p_+), \tag{3.4}$$

where $z = (p_K - p_\pi)^2/m_K^2$, and the general form factor $W_+(z)$ has been shown in ref. [30]. The piece induced by the EMO will interfere with the imaginary part of $W_+(z)$, which arises from the two-pion intermediate state [30]. The asymmetry is then written as

$$\left(\frac{\delta\Gamma}{2\Gamma}\right)_\ell^{\mathrm{EMO}} = \frac{|\Gamma(K^+ \to \pi^+\ell^+\ell^-) - \Gamma(K^- \to \pi^-\ell^+\ell^-)|_{\mathrm{EMO}}}{\Gamma(K^+ \to \pi^+\ell^+\ell^-) + \Gamma(K^- \to \pi^-\ell^+\ell^-)}. \tag{3.5}$$

Interestingly, with a kinematical cut $z \geq 4m_\pi^2/m_K^2$, the charge asymmetry in eq. (3.5) could be substantially enhanced [16]. Thus from eqs. (3.4) and (3.5), and using the upper bound of $|\,\mathrm{Im}\, C_\gamma^+|$ given in eq. (3.3), we can find the charge asymmetry for $\ell = e,\ \mu$ as

$$\left(\frac{\delta\Gamma}{2\Gamma}\right)_e^{\mathrm{EMO}} < 1.3 \times 10^{-4}, \qquad \left(\frac{\delta\Gamma}{2\Gamma}\right)_\mu^{\mathrm{EMO}} < 4.5 \times 10^{-4} \tag{3.6}$$

without the kinematical cut for z, and

$$\left(\frac{\delta\Gamma}{2\Gamma}\right)_e^{\mathrm{EMO}} < 1.2 \times 10^{-3}, \qquad \left(\frac{\delta\Gamma}{2\Gamma}\right)_\mu^{\mathrm{EMO}} < 1.3 \times 10^{-3} \tag{3.7}$$

with the cut $z \geq 4m_\pi^2/m_K^2$. Note that, differently from ref. [16], here we only use the experimental bound of $\mathrm{Br}(K_L \to \pi^0 e^+ e^-)$ to estimate the charge asymmetry in both electron and muon mode. So we are neglecting possible lepton-family violations.

4. Limits on new flavour structures

From eq. (2.12), one can get $\mathrm{Im}\, C_\gamma^+$ in the SM

$$\left|\mathrm{Im}\, C_\gamma^+\right|^{\mathrm{SM}} = \frac{3G_F}{\sqrt{2}}(m_s + m_d)|\,\mathrm{Im}\,\lambda_t C_{12}|\,. \tag{4.1}$$

Due to the smallness of $\mathrm{Im}\,\lambda_t \sim 10^{-4}$, this contribution from the SM is strongly suppressed, and far smaller than the upper bound (3.3). Therefore in the following we turn our attention to physics beyond the SM.

Among the possible new physics scenarios, low energy supersymmetry (SUSY) [31], represents one of the most interesting and consistent extensions of the SM. In generic supersymmetric models, the large number of new particles carrying flavour quantum numbers would naturally lead to large effects in CP violation and FCNC amplitudes [32]. Particularly, one can generate the enhancement of $C_{\gamma,g}^\pm$ at one-loop, via intermediate squarks and gluinos, which is due both to the strong coupling constant and to the removal of chirality suppression present in the SM. Full expressions for the Wilson coefficients generated by gluino exchange at the SUSY scale can be found in ref. [33]. We are interested here only in the contributions proportional to $m_{\tilde{g}}$, which are given by

$$C_{\gamma,\mathrm{SUSY}}^\pm(m_{\tilde{g}}) = \frac{\pi\alpha_s(m_{\tilde{g}})}{m_{\tilde{g}}}\left[(\delta_{\mathrm{LR}}^{\mathrm{D}})_{21} \pm (\delta_{\mathrm{LR}}^{\mathrm{D}})_{12}^*\right] F_{\mathrm{SUSY}}(x_{gq})\,, \tag{4.2}$$

$$C_{g,\mathrm{SUSY}}^\pm(m_{\tilde{g}}) = \frac{\pi\alpha_s(m_{\tilde{g}})}{m_{\tilde{g}}}\left[(\delta_{\mathrm{LR}}^{\mathrm{D}})_{21} \pm (\delta_{\mathrm{LR}}^{\mathrm{D}})_{12}^*\right] G_{\mathrm{SUSY}}(x_{gq})\,, \tag{4.3}$$

where $(\delta_{\mathrm{LR}}^{\mathrm{D}})_{ij} = (M_{\mathrm{D}}^2)_{i_{\mathrm{L}}j_{\mathrm{R}}}/m_{\tilde{q}}^2$ denotes the off-diagonal entries of the (down-type) squark mass matrix in the super-CKM basis, $x_{gq} = m_{\tilde{g}}^2/m_{\tilde{q}}^2$ with $m_{\tilde{g}}$ being the average gluino mass and $m_{\tilde{q}}$ the average squark mass. The explicit expressions of $F_{\mathrm{SUSY}}(x)$ and $G_{\mathrm{SUSY}}(x)$ are given in ref. [2], but noting that they do not depend strongly on x, it is sufficient, for our purposes, to approximate $F_{\mathrm{SUSY}}(x) \sim F_{\mathrm{SUSY}}(1) = 2/9$ and $G_{\mathrm{SUSY}}(x) \sim G_{\mathrm{SUSY}}(1) = -5/18$. In any case it will be easy to extend the numerology once x_{gq} is better known. Also the determination of the Wilson coefficients in eqs. (4.2) and (4.3) can be improved by the renormalization group analysis [2, 6]. Then by taking $m_{\tilde{g}} = 500\,\mathrm{GeV}$, $m_t = 174\,\mathrm{GeV}$, $m_b = 5\,\mathrm{GeV}$, and $\mu = m_c = 1.25\,\mathrm{GeV}$, we will have

$$\left|\mathrm{Im}\, C_\gamma^+\right|^{\mathrm{SUSY}} = 2.4 \times 10^{-4}\,\mathrm{GeV}^{-1}\left|\mathrm{Im}\left[\left(\delta_{\mathrm{LR}}^{\mathrm{D}}\right)_{21} + \left(\delta_{\mathrm{LR}}^{\mathrm{D}}\right)_{12}^*\right]\right|. \tag{4.4}$$

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From eq. (3.3), we obtain

$$\left|\mathrm{Im}\left[\left(\delta^{\mathrm{D}}_{\mathrm{LR}}\right)_{21}+\left(\delta^{\mathrm{D}}_{\mathrm{LR}}\right)^{*}_{12}\right]\right|<7.7\times10^{-4}\,, \tag{4.5}$$

comparable with the one given by the lattice calculation [6].

5. Conclusions

To conclude, supersymmetric extensions of the SM may enhance the Wilson coefficients of the electromagnetic penguin operators. This leads to interesting phenomenology to be studied: the direct CP violation in $K_L \to \pi^0 e^+ e^-$ and the charge asymmetry in $K^\pm \to \pi^\pm \ell^+ \ell^-$. To this purpose we evaluate the relevant matrix element in the χQM. Interestingly we find a very good agreement with lattice results for the natural parameters of the model [6]. The present experimental upper bound of $\mathrm{Br}(K_L \to \pi^0 e^+ e^-)$ allows to obtain an upper bound of $|\,\mathrm{Im}\, C_\gamma^+|$, and thus to predict the upper bound of the charge asymmetry in $K^\pm \to \pi^\pm \ell^+ \ell^-$ induced by EMO. The analysis shows that the predictions for the relevant matrix elements are solid and thus high precision measurements of CP-observables might probe interesting extensions of the SM.

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A. Appendix

Here we present the derivation for eq. (2.11) in the χQM. Including the constituent quark mass term in eq. (2.1), the strong lagrangian in the rotated basis (eq. (2.4)) and in the euclidean space is (after we switch off contributions from the EMO)

$$\mathcal{L}^E_{\mathrm{Str}} = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \bar{Q} D_E Q\,, \tag{A.1}$$

where $G^a_{\mu\nu}$ is the gluon fields strength tensor, and D_E the euclidean Dirac operator

$$D_E = \gamma_\mu \nabla_\mu + M = \gamma_\mu(\partial_\mu + \mathcal{A}_\mu) + M\,, \tag{A.2}$$

with

$$\mathcal{A}_\mu = iG_\mu + \Gamma_\mu - \frac{i}{2}\gamma_5 \xi_\mu\,, \qquad M = -\frac{1}{2}(\Sigma - \gamma_5 \Delta) - M_Q\,. \tag{A.3}$$

Note that, in the present paper, we use the same notations as in ref. [20] and so for the euclidean quantities, $\gamma_\mu^+ = \gamma_\mu$, $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$, and $\sigma_{\mu\nu} = -i/2[\gamma_\mu, \gamma_\nu]$. The external vector and axial-vector fields now appear in Γ_μ and ξ_μ

$$\Gamma_\mu = \frac{1}{2}\left[\xi^+(\partial_\mu - ir_\mu)\xi + \xi(\partial_\mu - il_\mu)\xi^+\right]\,, \tag{A.4}$$

$$\xi_\mu = i\left[\xi^+(\partial_\mu - ir_\mu)\xi - \xi(\partial_\mu - il_\mu)\xi^+\right]\,, \tag{A.5}$$

and

$$\Sigma = \xi^+ \mathcal{M} \xi^+ + \xi \mathcal{M} \xi \,, \qquad \Delta = \xi^+ \mathcal{M} \xi^+ - \xi \mathcal{M} \xi \,. \tag{A.6}$$

Here $\mathcal{M}$ is the current quark mass matrix, and

$$\Gamma_\mu^+ = -\Gamma_\mu, \qquad \xi_\mu^+ = \xi_\mu \,, \qquad \Sigma^+ = \Sigma \,, \qquad \Delta^+ = -\Delta \,, \qquad \mathcal{M}^+ = \mathcal{M} \,. \tag{A.7}$$

The Σ- and Δ-terms break chiral symmetry explicitly.

The euclidean effective action $W_E(U, r, l, \mathcal{M}, M_Q)$ is obtained as follows

$$\exp W_E(U, r, l, \mathcal{M}, M_Q) = \frac{1}{Z} \int \mathcal{D} G_\mu \exp \left(-\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} \right) \exp \Gamma_E(\mathcal{A}, M) \,, \tag{A.8}$$

where Z is the normalization factor, and

$$\exp \Gamma_E(\mathcal{A}, M) = \int \mathcal{D}\bar{Q} \mathcal{D} Q \exp \int d^4x \bar{Q} D_E Q = \det D_E \,. \tag{A.9}$$

Since we are concerned with the non-anomalous part of the effective action, we have

$$\Gamma_E(\mathcal{A}, M) = \frac{1}{2} \ln \det D_E^+ D_E \,, \tag{A.10}$$

with

$$D_E^+ = -\gamma_\mu \left(\partial_\mu + i G_\mu + \Gamma_\mu + \frac{i}{2} \gamma_5 \xi_\mu \right) - \frac{1}{2} (\Sigma + \gamma_5 \Delta) - M_Q \,. \tag{A.11}$$

Using the technique of the heat kernel expansion [25], one can derive the effective strong lagrangian starting from (A.10), which has been discussed extensively in the literature.

Now we switch on the EMO. Note that this operator has been expressed using the rotated basis in eq. (2.9); it is thus easy to know that (A.10) should become

$$\Gamma_E(\mathcal{A}, M) = \frac{1}{2} \ln \det D_E^{+\prime} D_E{}' \,, \tag{A.12}$$

with

$$D_E{}' = D_E + J \,, \qquad D_E^{+\prime} = D_E^+ + J^+ \,, \tag{A.13}$$

$$J = \sigma_{\mu\nu} J_{\mu\nu} \,, \qquad J^+ = \sigma_{\mu\nu} J^+_{\mu\nu} \,, \tag{A.14}$$

and

$$\begin{aligned} J_{\mu\nu} &= -\left(\frac{1-\gamma_5}{2} \xi^+ \lambda \xi^+ m_s + \frac{1+\gamma_5}{2} \xi \lambda \xi m_d \right) C_{\text{EMO}} F_{\mu\nu} - \\ &\quad - \left(\frac{1+\gamma_5}{2} \xi \lambda^+ \xi m_s + \frac{1-\gamma_5}{2} \xi^+ \lambda^+ \xi^+ m_d \right) C^*_{\text{EMO}} F_{\mu\nu} \,, \\ J^+_{\mu\nu} &= J_{\mu\nu}(\gamma_5 \leftrightarrow -\gamma_5) \,. \end{aligned} \tag{A.15}$$

Thus, one can get

$$D_E^{+\prime} D_E{}' - M_Q^2 = -\nabla_\mu \nabla_\mu + E + F_1 + F_2 \,, \tag{A.16}$$

with

$$E = iM_Q\gamma_\mu\gamma_5\xi_\mu - \frac{i}{2}\sigma_{\mu\nu}R_{\mu\nu}\,, \tag{A.17}$$

$$F_1 = -\gamma_\mu\sigma_{\alpha\beta}d_\mu J_{\alpha\beta} + \frac{i}{2}\gamma_\mu\sigma_{\alpha\beta}\left\{\gamma_5\xi_\mu, J_{\alpha\beta}\right\} - M_Q\sigma_{\mu\nu}\left(J_{\mu\nu} + J^+_{\mu\nu}\right), \tag{A.18}$$

$$F_2 = -4i\gamma_\mu J_{\mu\nu}\nabla_\nu\,, \tag{A.19}$$

and

$$R_{\mu\nu} = iG_{\mu\nu} - i\left(\frac{1+\gamma_5}{2}\xi^+F_{R\mu\nu}\xi + \frac{1-\gamma_5}{2}\xi F_{L\mu\nu}\xi^+\right). \tag{A.20}$$

Here we set $\Sigma = \Delta = 0$, d_μ is the covariant derivative with respect to the Γ_μ-connection, i.e. $d_\mu A = \partial_\mu A + [\Gamma_\mu, A]$, and the relation

$$[\gamma_\mu, \sigma_{\alpha\beta}] = 2i\left(\delta_{\mu\alpha}\gamma_\beta - \delta_{\mu\beta}\gamma_\alpha\right) \tag{A.21}$$

has been used. We only include the linear terms of $J_{\mu\nu}$ in F_1 and F_2 because we are concerned about the $O(G_F)$ $\Delta S = 1$ transitions.

Starting from (A.12), and in terms of the proper time method [25], we have

$$\Gamma_E(\mathcal{A}, M) = -\frac{1}{2}\int d^4x\,\mathrm{Tr}\int_0^\infty \frac{d\tau}{\tau}\left\langle x\left|\exp\left(-\tau D_E^{+\prime}D_E{}'\right)\right|x\right\rangle, \tag{A.22}$$

where the trace is taken in colour, flavour, and Lorentz space. By inserting a complete set of plane waves and using (A.16), we obtain

$$\Gamma_E(\mathcal{A}, M) = -\frac{1}{2}\int d^4x\,\mathrm{Tr}\int_0^\infty \frac{d\tau}{\tau}\int \frac{d^dp_E}{(2\pi)^d}\exp\left[-\tau\left(p_E^2 + M_Q^2\right)\right]\exp(-\tau\mathcal{D}')\,, \tag{A.23}$$

where

$$\mathcal{D}' = E - \nabla\cdot\nabla + F_1 - 2ip_E\cdot\nabla + F_2 + 4\gamma_\mu p_{E\nu}J_{\mu\nu}\,. \tag{A.24}$$

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